

# MEASUREMENT OF THE NONLINEAR MOMENTUM COMPACTION FACTOR IN RHIC \*

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## Abstract

During gold beam acceleration in the Relativistic Heavy Ion Collider (RHIC), the transition energy has to be crossed at  $\gamma_t \approx 23$ . Since close to  $\gamma_t$  the longitudinal slip factor  $\gamma_t^{-2} - \gamma^{-2}$  becomes very small, the longitudinal momentum compaction factor  $\alpha_1$  becomes significant. Measurements of this factor using longitudinal phase space tomography will be reported.

## INTRODUCTION

The Relativistic Heavy Ion Collider (RHIC) consists of two superconducting storage rings, capable of accelerating hadron beams from protons to fully stripped gold ions up to energies of 100 GeV/nucleon in the case of gold. Ion species other than protons have to cross transition energy around  $\gamma_t = 23.2$  during acceleration in RHIC, which is accomplished by a set of  $\gamma_t$ -quadrupoles equipped with special power supplies that can switch the sign of the magnetic field within 30 msec, thus providing a  $\gamma_t$  jump of  $\Delta\gamma_t = 1.0$ .

To minimize longitudinal emittance blow-up during the  $\gamma_t$  jump due to bucket mismatch, a detailed understanding of the beam dynamics is required. Here we report an attempt to measure the nonlinear momentum compaction factor  $\alpha_1$  using tomographic phase space reconstruction.

## TOMOGRAPHIC PHASE SPACE RECONSTRUCTION

To fully reconstruct the  $n$ -dimensional picture of an object, tomography requires a set of  $n - 1$ -dimensional projections of this object, taken at different angles spanning at least 180 degrees. In the case of tomographic reconstruction of the longitudinal phase space in a storage ring, this rotation is provided by phase space dynamics. However, this dynamics is not just a simple rotation of a rigid object, but is intrinsically nonlinear with the rotation frequency (synchrotron frequency) being a function of the phase space amplitude. This difficulty can be overcome by taking into account the exact equations of motion which can be arbitrarily complex [1].

Taking the exact equations of motion, a set of test particles launched on a regular grid in phase space are tracked and sorted into  $N_{\text{bins}}$  bins that correspond to the binning of the

measured profiles each time a longitudinal bunch profile  $i$  was obtained by the wall current monitor. Each particle  $k$  is therefore assigned a bin number  $N_{i,j}^{\text{bin}}(k)$ . Next, the number  $N_{i,j}^{\text{population}}$  of test particles in the  $j$ th bin at the time the  $i$ th profile was taken is determined.

During the reconstruction process an intensity  $i$  is assigned to each test particle by an iterative back-projection algorithm according to the measured bunch profiles. This process increases the intensity  $I_k$  of all test particles that fall into a certain profile bin  $j$  at a specific time when the  $i$ th profile was taken by

$$\Delta I_{k: N_{\text{bin}}(k)=j} = \frac{1}{N_{\text{profiles}} N_{i,j}^{\text{population}}} \sum_{i=0}^{N_{\text{profiles}}} h_{i,j}^{\text{meas}}, \quad (1)$$

where  $h_{i,j}^{\text{meas}}$  is the measured profile height of the  $j$ th bin in the  $i$ th profile, and  $N_{\text{profiles}}$  is the total number of profiles used for the reconstruction.

When this has been done for all profiles, the algorithm calculates the projections of the resulting distribution that correspond to the profiles measured by the wall current monitor. The difference between measured and reconstructed profiles is then iteratively back-projected.

The remaining discrepancy between measured and reconstructed profiles after a fixed number of iterations is then used as a quantitative measure of the quality of the reconstruction, which allows for parameter fitting [1].

## THE NONLINEAR MOMENTUM COMPACTION FACTOR

The frequency-slip factor  $\eta$  which characterizes the chromatic behavior in the longitudinal phase space is defined as the relative change of revolution frequency  $\omega$  per unit change of the relative momentum  $\delta = \Delta p/p$ ,

$$\eta = -\frac{1}{\omega_s} \frac{d\omega}{d\delta}. \quad (2)$$

Here  $\omega_s$  denotes the revolution frequency of the synchronous particle.

In general, the slip-factor  $\eta$  is a nonlinear function of  $\delta$ ,

$$\eta = \eta_0 + \eta_1 \delta + \mathcal{O}(\delta^2), \quad (3)$$

with [2]

$$\eta_0 = \alpha_0 - \frac{1}{\gamma_s^2}, \quad (4)$$

$$\eta_1 = 2\alpha_0\alpha_1 - 2\eta_0\alpha_0 + 3\frac{\beta_2^2}{\gamma_s^2}, \quad (5)$$

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where  $\gamma_s = (1 - \beta_s^2)^{-1/2}$  is the Lorentz factor of the synchronous particle,  $\beta_s = v_s/c$ , and  $\alpha_0 = 1/\gamma_t^2$ . In the vicinity of  $\gamma_t$ ,  $1/\gamma_s^2 = 1/\gamma_t^2 = \alpha_0$ , and therefore

$$\eta_1 = 2\frac{\alpha_1}{\gamma_t^2} + 3\frac{\beta_s^2}{\gamma_t^2}. \quad (6)$$

While the linear part  $\eta_0$  of the slip factor  $\eta$  changes sign when the transition energy is crossed, the nonlinear contribution  $\eta_1$  does not. This effect leads to bucket mismatch at the transition jump unless  $\alpha_1$  can be specifically chosen. With  $\beta_s = 1$  for relativistic beams, the contribution of  $\eta_1$  vanishes for  $\alpha_1 = -3/2$ .

## SIMULATIONS

To test the feasibility of measuring the nonlinear momentum compaction factor  $\alpha_1$  tomographically, simulations were performed. A set of 100000 particles with gaussian distributions in phase  $\phi$  and energy deviation  $\delta$  were tracked using the parameter setpoints as given in Table 1. To potentially improve the convergence of the subsequent parameter fit, a quadrupole oscillation was induced by launching the particles with a deliberate bucket mismatch, namely  $\sigma_\delta = 0.2 \cdot \delta_{\max}$  and  $\sigma_\phi = 0.1 \cdot \phi_{\max} = 0.1 \cdot \pi$ , where  $\delta_{\max}$  denotes the bucket height. Projections (wall current monitor profiles) of the evolving distribution were calculated every 125 turns.

First, a one-parameter fit for the nonlinear momentum compaction factor  $\alpha_1$  was performed, assuming all other parameters as exactly known. As shown in Figure 1, several local minima exist in the vicinity of the correct value of  $\alpha_1 = 0.35$ , which may be explained by the limited number of particles used to generate the profiles, and/or the binning of those profile data. Smoothing the curve shown in Figure 1 by regarding those small fluctuations as some sort of noise results in a nonlinear momentum compaction factor around  $\alpha_1 \approx -0.8$ , which significantly differs from the correct value.

In a second step, the profiles were used to reconstruct the initial phase space distribution, simultaneously fitting for three unknown parameters as it is required in the case with measured data, the nonlinear momentum compaction factor  $\alpha_1$ , the RF voltage  $U_{\text{RF}}$ , and the bin position of the synchronous phase in the profiles using a simulated annealing technique [3, 4].

The nonlinear momentum compaction factor was fitted as  $\alpha_1 = 0.32 \pm 0.14$ , where the error is taken as the rms deviation from the average over several fitting runs with different initial parameters. This showed that phase space tomography together with simultaneous fitting of the three unknown parameters may indeed be a feasible method to measure the nonlinear momentum compaction factor  $\alpha_1$ .

## MEASUREMENTS

For the measurements, two RF voltage jumps were introduced 1.0 sec and 0.5 sec before the transition jump to

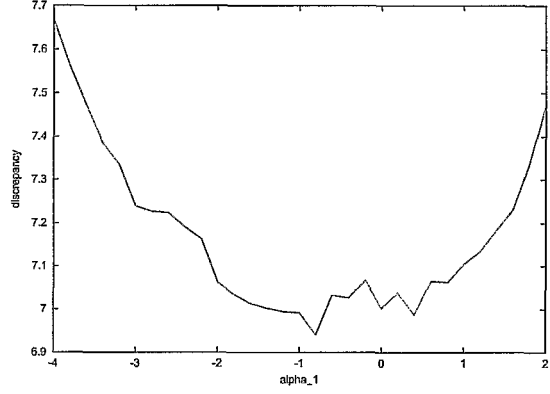


Figure 1: Remaining discrepancy as function of  $\alpha_1$ , using simulated data. The RF voltage is set to the exact value during this one-parameter fit.

$\gamma_t$	23.7647
$\gamma_i$	22.85775
$d\gamma/dt$	0.4556/sec
$U_{\text{RF}}$	145 kV
$n_{\text{harm}}$	360
$\alpha_1$	0.35

Table 1: Parameter table for the simulation test. All parameters are set according to the situation in the “blue” RHIC ring.  $\alpha_1$  is chosen according to model calculations with both chromaticities set to  $\xi_{x,y} = -2$ .

create some longitudinal quadrupole oscillation, as shown in Figure 2. 50 profiles taken every 125 turns were used for the  $\alpha_1$  measurement, starting 0.1 seconds after the second RF jump.

As in the simulation test, a simulated annealing technique was applied to simultaneously fit for the three unknown parameters  $\alpha_1$ ,  $U_{\text{RF}}$ , and the position of the synchronous phase in the wall current monitor profiles. The resulting value for the nonlinear momentum compaction factor is  $\alpha_1 = -1.37 \pm 0.11$ , which significantly differs from the value calculated by the model,  $\alpha_1 = 0.35$ .

This discrepancy may have several causes. First of all, tomographic phase space reconstruction may not be sensitive enough to changes of  $\alpha_1$  on the order of  $\pm 1$ . This explanation is backed by a similar observation during the simulation tests, where  $\alpha_1$  was found with a similar discrepancy during one-parameter fits.

A poor sensitivity of the resulting discrepancy to small changes in  $\alpha_1$  leads to additional problems during multi-parameter fits. As Figure 3 shows, different values of  $\alpha_1$  result in different local minima for different setpoints of the RF voltage during the reconstruction. In the vicinity of  $\alpha_1 \pm 1$ , the remaining discrepancy is clearly dominated by fluctuations, which may be interpreted as noise.

Finally, the nonlinear momentum compaction factor de-

BLUE bunch length around transition, fill 02797

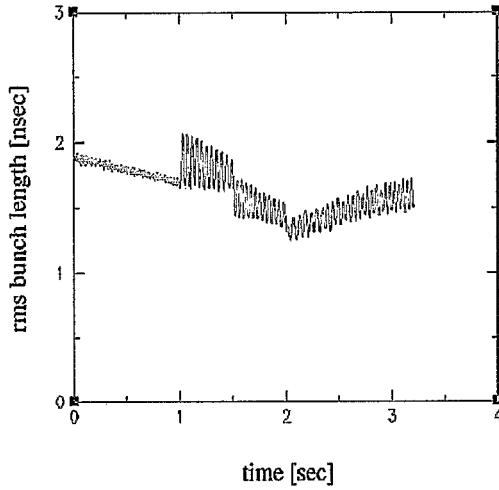


Figure 2: Measured bunch length with two RF voltage jumps (at 1.0 sec and 1.5 sec, respectively) to induce a quadrupole oscillation. The transition jump occurs at 2.0 sec, indicated by the minimum bunch length.

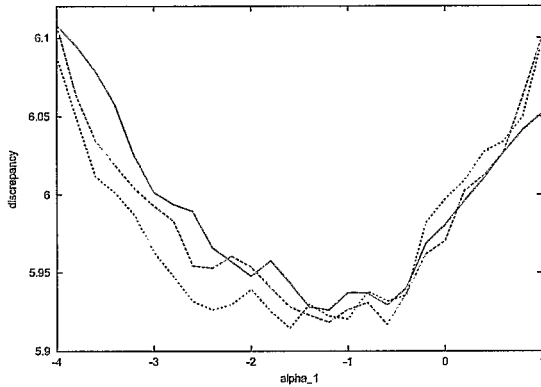


Figure 3: Remaining discrepancy as function of  $\alpha_1$  for different RF voltages. The red line corresponds to  $U_{RF} = 140$  kV, the green one to  $U_{RF} = 145$  kV, and the blue one to  $U_{RF} = 150$  kV.

depends on various machine parameters such as the chromaticity, as Figure 4 shows. Since the superconducting RHIC dipoles have a non-negligible field-dependent sextupole component, exact modeling of the machine is not trivial. This sextupole component contributes to the chromaticity, leading to a different setting of the regular sextupoles than in an ideal machine without sextupole contributions from the dipoles. Furthermore, the sextupole component of the dipoles also results in tune change due to an overall radial orbit shift of about 1.3 mm, which in turn requires adjustment of the tune quadrupoles to keep the tunes at their target values [5]. All these corrections to the ideal

model contribute to changes in the nonlinear momentum compaction factor.

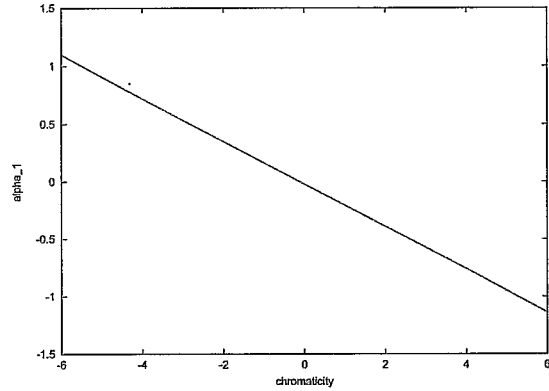


Figure 4: Nonlinear momentum compaction factor  $\alpha_1$  as calculated from the model, for different chromaticities,  $\xi_x = \xi_y$ .

## CONCLUSION

An alternative approach to measuring the nonlinear momentum compaction factor  $\alpha_1$  of RHIC has been made, namely the use of tomographic reconstruction of the longitudinal phase space. The discrepancies with the model may be due to discrepancies between the model and the real machine itself as well as due to insufficient sensitivity of the quality factor of those parameters fits to small changes in  $\alpha_1$ .

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